Warm Up

List the equations for all vertical and horizontal asymptotes

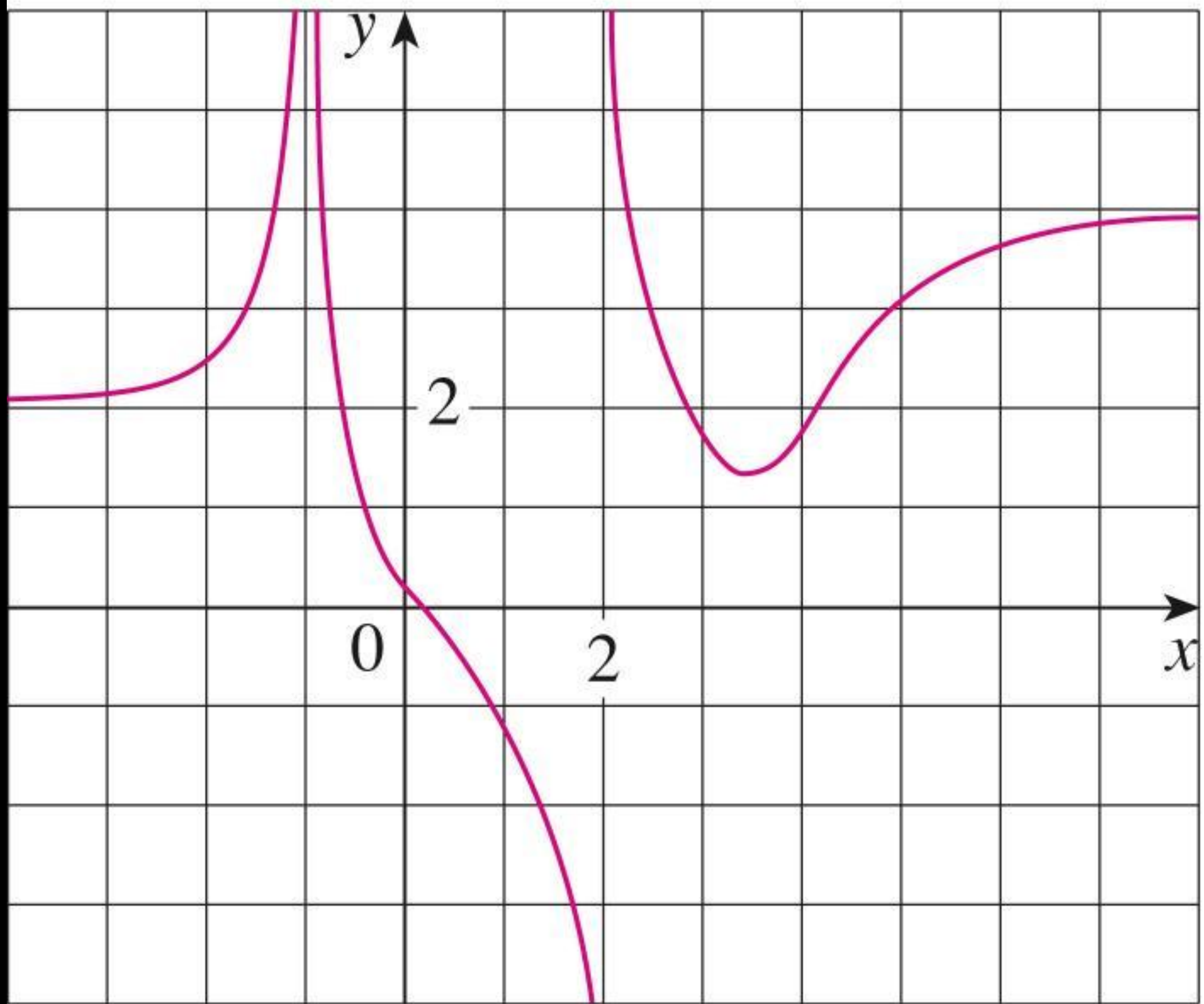


FIGURE 5

Vertical Asymptotes: (Recall 2.2)

The line $x = a$ is called a vertical asymptote of the curve y = f(x) if at least one of the following six statements is true.

$$\lim_{x\to a} f(x) = \infty \qquad \lim_{x\to a^-} f(x) = \infty \qquad \lim_{x\to a^+} f(x) = \infty$$

$$\lim_{x\to a} f(x) = -\infty \qquad \lim_{x\to a^-} f(x) = -\infty \qquad \lim_{x\to a^+} f(x) = -\infty$$

Horizontal Limits (Limits at Infinity)

The line y = L is called a horizontal asymptote of the curve y = f(x) if either

$$\lim_{x\to\infty} f(x) = L \quad or \quad \lim_{x\to-\infty} f(x) = L$$

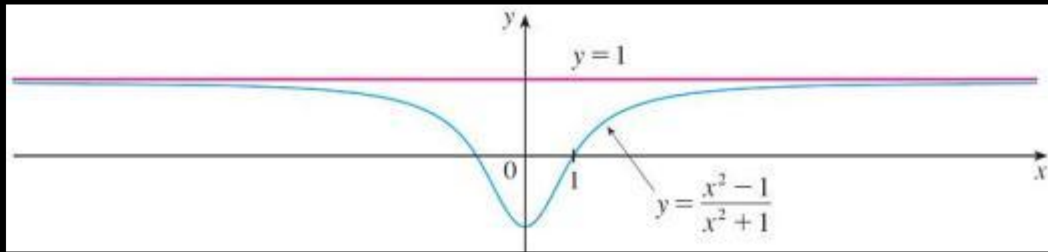
- *A calculus type of question would be*

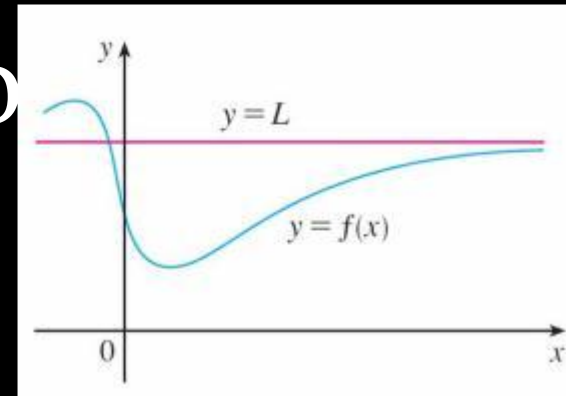
$$\lim_{x \to 1} \frac{3x - 3}{x^2 - 1}$$

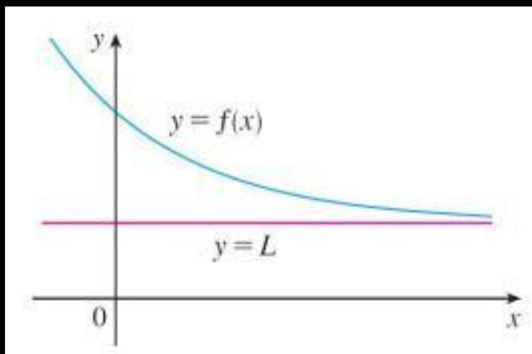
- *Always try direct substitution. For this it would not work though.*
- *So now consider what is happening to the graph at x = 1. There some type of discontinuity (Hole, Jump, Asymptote). We rule out hole, so let's see if there is an asymptote.*
- *Consider* $y = \frac{3x-3}{x^2-1}$
- Write an equation for the vertical asymptote(s) of the graph.
- After simplifying the function we see that x = 1 is a vertical asymptote.
- So then it's a matter of determining if the limit is $\infty, -\infty, or\ does\ not\ exist$. I went over how to do this without a calculator in class. We will go over again.

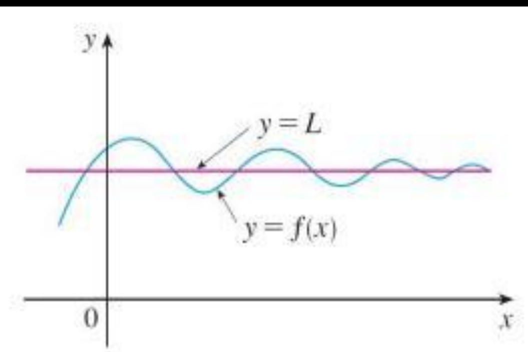
Horizontal Asymptotes

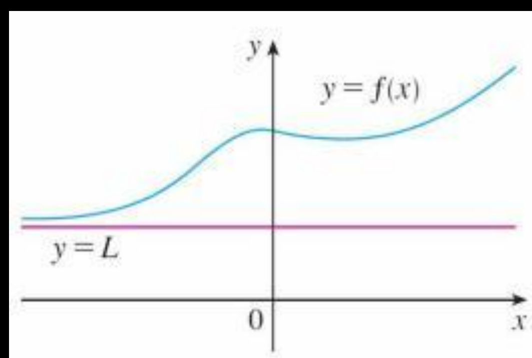
$$\lim_{x \to \infty} f(x) = L \quad or \quad \lim_{x \to -\infty} f(x) = L$$

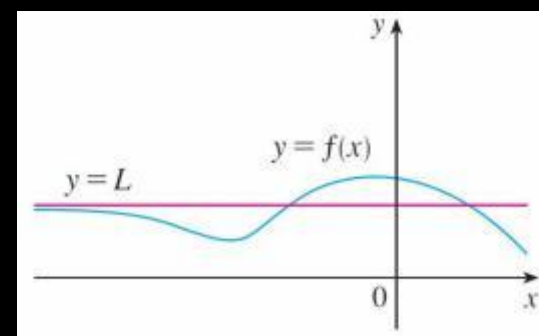












3 **Definition** The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = L \qquad \text{or} \qquad \lim_{x \to -\infty} f(x) = L$$

What you will need to get out of this:

- With Horizontal Asymptotes (as we look at x approaching infinity or negative infinity)
  – If it's NOT a rational function
    - Use intuition
    - Use knowing the function and it's graph
  – If it IS a rational function
    - Use what you know about finding horizontal asymptotes from Algebra 2/MA
    - Find the highest degree x and divide everything by it.

Sometimes it's intuitive.

Example: Evaluate

$$\lim_{x \to \infty} \left(x + \sqrt{x^2 + 2x}\right)$$

Sometimes we can just think about what happens to the value of x as we get bigger and bigger.

$$\lim_{x \to \infty} x^3$$

$$\lim_{x \to -\infty} x^3$$

Sometimes it's knowing the function

$$\lim_{x \to -\infty} \tan x$$

Tangent is a periodic function so as we go further into the negatives, we do not have a limit. It does not exist.

$$\lim_{x \to \infty} x^2(x^2 - 1)^2(x + 2)$$

This is a 7th degree polynomial with a leading coefficient that is positive. So, study up on what graphs looks like (end behavior wise) of polynomials: http://www.purplemath.com/modules/polyends.htm

Sometimes we need to utilize algebra, simplification techniques and this idea to the right.

$$\lim_{x \to \infty} \frac{1}{x^r}$$

If $r > 0$ is a rational number then what could this limit be?

Example: Evaluate $\lim_{x\to\infty} \frac{4x^3+3x+1}{7x^3+2x^2+x}$

We do these by dividing EVERYTHING in the fraction by the highest degree x. For example in the second one we will divide everything by x^3.

$$\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1}$$

See Example 3 on page 133

$$\lim_{x\to\infty} \frac{t-t\sqrt{t}}{2t^{3/2}+3t-5}$$

$$\lim_{x \to \infty} \frac{x^2 + x}{3 - x}$$

NEXT CLASS

- Give them time to practice 2.6
- Review the concepts
- AP style quiz
- 2.7
- Then review
- Then test
- HW tonight