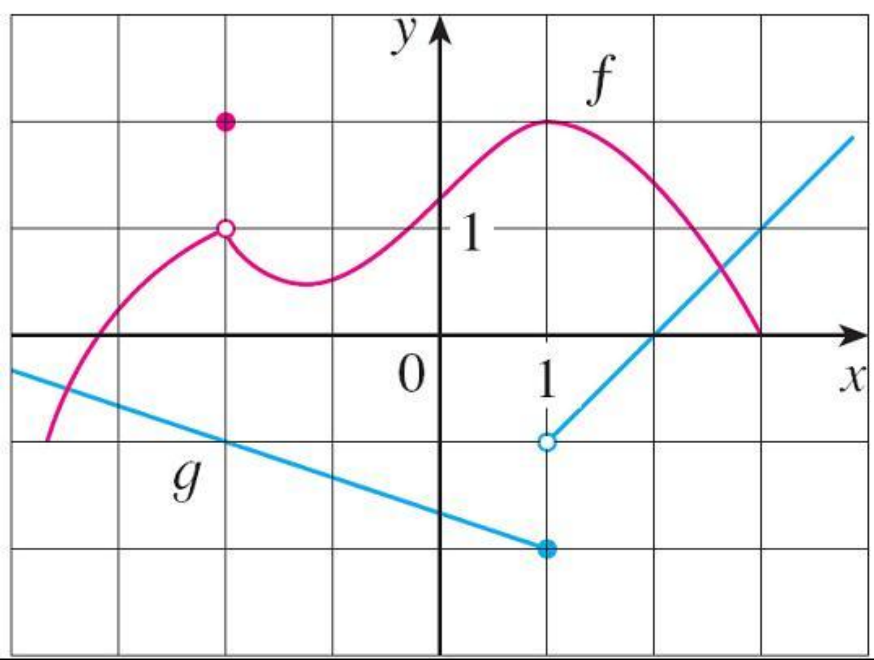
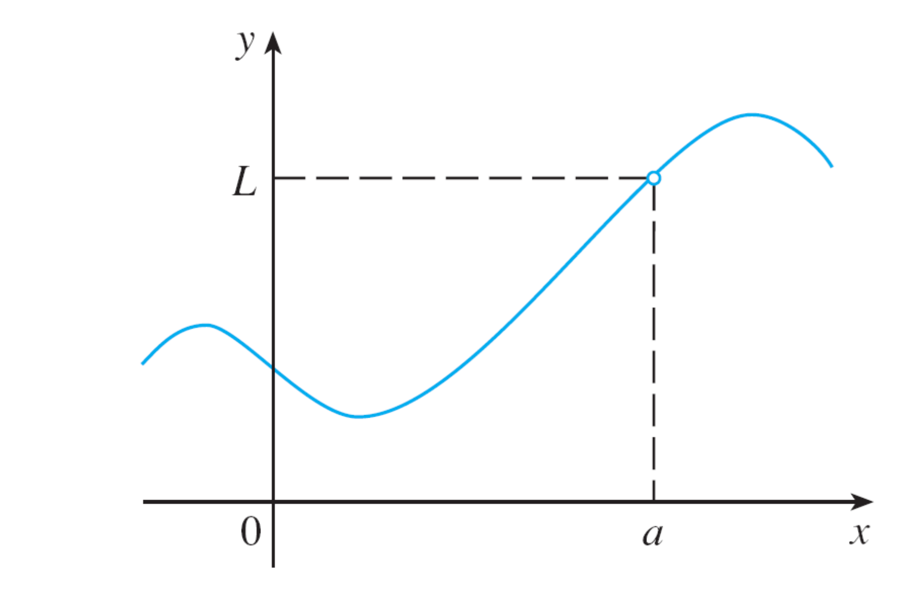
2.4-2.5

The intuitive definition of a limit that we have learned about: *looking in the hood of a value* a.

Is inadequate for some purposes because such phrases as: "x is close to 2" and "f(x) gets closer and closer to L" are vague.





(c)

Geometrically

Geometrically, this statement means that if any small interval $(L - \varepsilon, L + \varepsilon)$ is given around L, then we can find an interval $(a - \delta, a + \delta)$ around a such that f maps all the points in the interval $(a - \delta, a + \delta)$ to the interval $(L - \varepsilon, L + \varepsilon)$

In motion file 1 epsilon delta
1.5 Can you find a delta that would work?
2 Can't find a delta for this epsilon that would work

$\boxed{2}$ **Definition** Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then we say that the **limit of $f(x)$ as x approaches a is L**, and we write

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \quad 0 < |x - a| < \delta \qquad \text{then} \qquad |f(x) - L| < \varepsilon$$

Infinite Limits

Infinite limits can also be defined in a precise

6 Definition Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that for every positive number M there is a positive number δ such that

$$\text{if} \qquad 0 < |x - a| < \delta \qquad \text{then} \qquad f(x) > M$$

Infinite Limits

This says that the values of $f(x)$ can be made arbitrarily large (larger than any given number M) by taking x close enough to a (within a distance δ, where δ depends on M, but with $x \neq a$). A geometric illustration is shown in Figure 10.

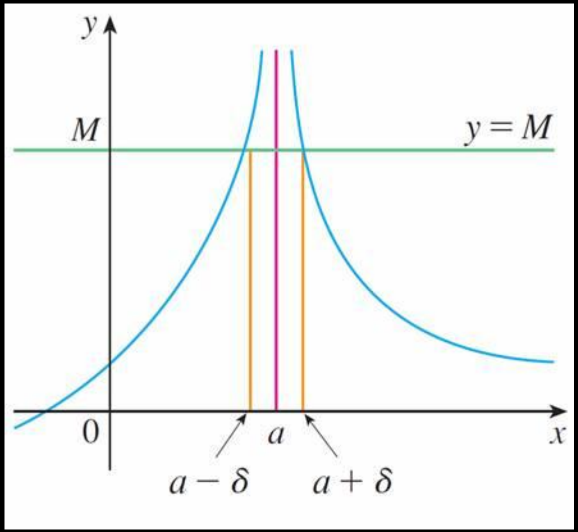


Figure 10

Infinite Limits

Given any horizontal line $y = M$, we can find a number $\delta > 0$ such that if we restrict to x lie in the interval $(a - \delta, a + \delta)$ but $x \neq a$, then the curve $y = f(x)$ lies above the line $y = M$.

You can see that if a larger M is chosen, then a smaller δ may be required.

Infinite Limits

7 Definition Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = -\infty$$

means that for every negative number N there is a positive number δ such that

$$\text{if} \qquad 0 < |x - a| < \delta \qquad \text{then} \qquad f(x) < N$$

Warm Up

Assume that $f(1) = -5$ and $f(3) = 5$. Does there have to be a value of x, between 1 and 3, such that $f(x) = 0$?

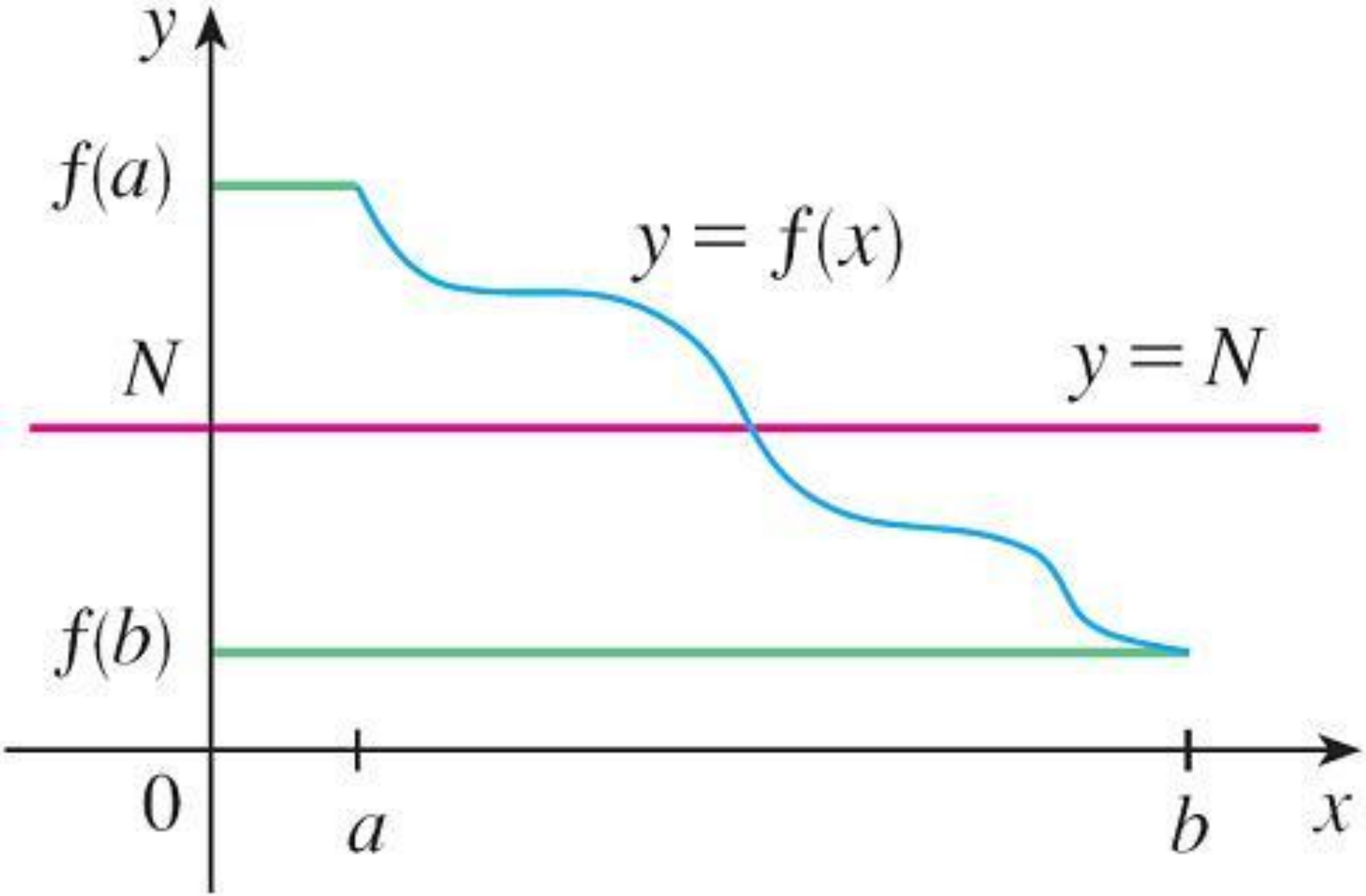
You might want to sketch a quick coordinate plane and plot the two points to visualize what it's asking.

Answer: No, there does not. Only if the function is continuous does the Intermediate Value Theorem then tell us there MUST be an x between 1 and 3 s.t. $f(x) = 0$

Today 2.5 Continuity

- Review Continuity at a point
- Continuity of a function
- Discontinuity
- Take questions on Worksheet and Epsilon/Delta stuff
- Work time.

10 **The Intermediate Value Theorem** Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.



Imagine a graph of *f, a continuous function.*

1. If the point (0, -2) and the point (2, 1) are on the function, does the graph cross the x axis?
2. Does the point (1, 0) have to be on *f*?

Continuity

A function f is continuous at a number a if :

a. $f(a)$ is defined (that is, a is in the domain of f)

b. $\lim_{x \to a} f(x)$ Exists

c. $\lim_{x \to a} f(x) = f(a)$

Can you come up with examples where a. holds but b. and c. do not?

Worksheet!

Discontinuity (vocab lesson)

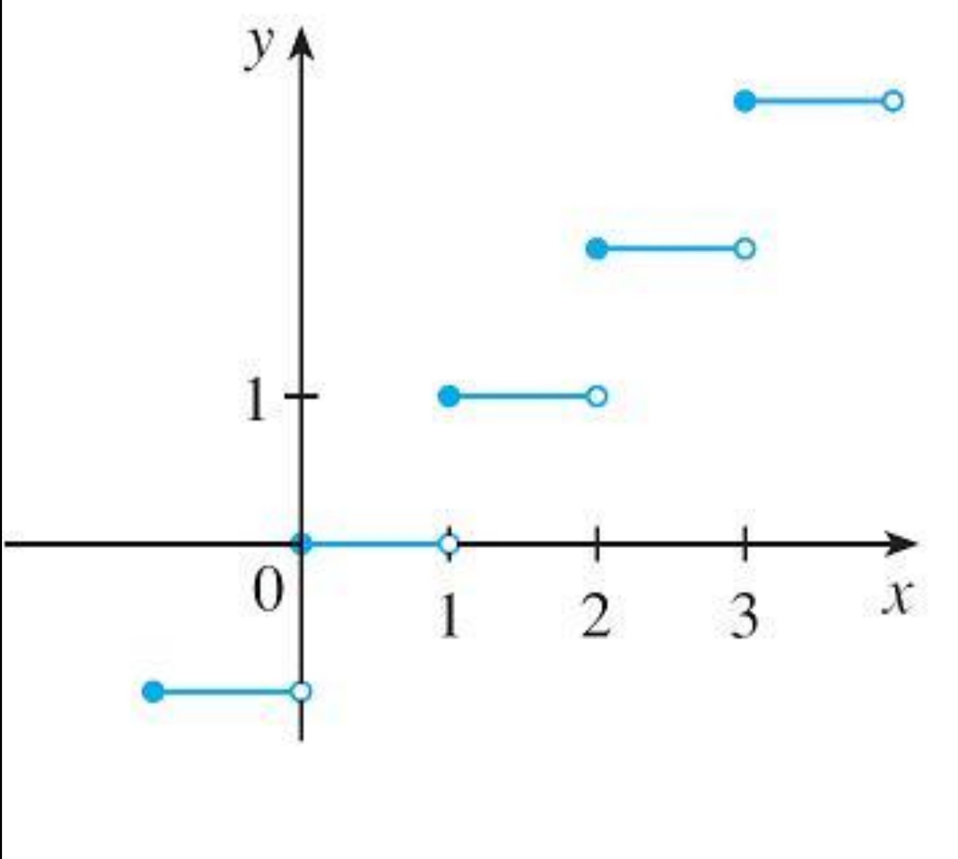
1. Discontinuous at a number a
   a. A discontinuity is removable if we could remove the discontinuity by redefining f at a single number
2. Infinite discontinuity if it occurs at a vertical asymptote
3. A discontinuity is a jump discontinuity if it occurs as a function “jumps” from one value to another.

Note we can say a function is continuous from the right/left at a

Greatest Integer Function

https://www.youtube.com/watch?v=k698XGE6EUA

http://mathworld.wolfram.com/FloorFunction.html



(d) $f(x) = [\![x]\!]$