Warm Up

In your notes answer this question. We will discuss it as a group.

1. What would the table look like for the inverse of the function below?
2. Evaluate $f(f^{-1}(5))$ and $f^{-1}(f(7))$
3. Consider the function $g(x) = 3x$, evaluate $g(g^{-1}(6))$

x	f(x)
1	-6
5	1
7	4
8	5
23	7

x	$f^{-1}(x)$
-6	1
1	5
4	7
5	8
7	23

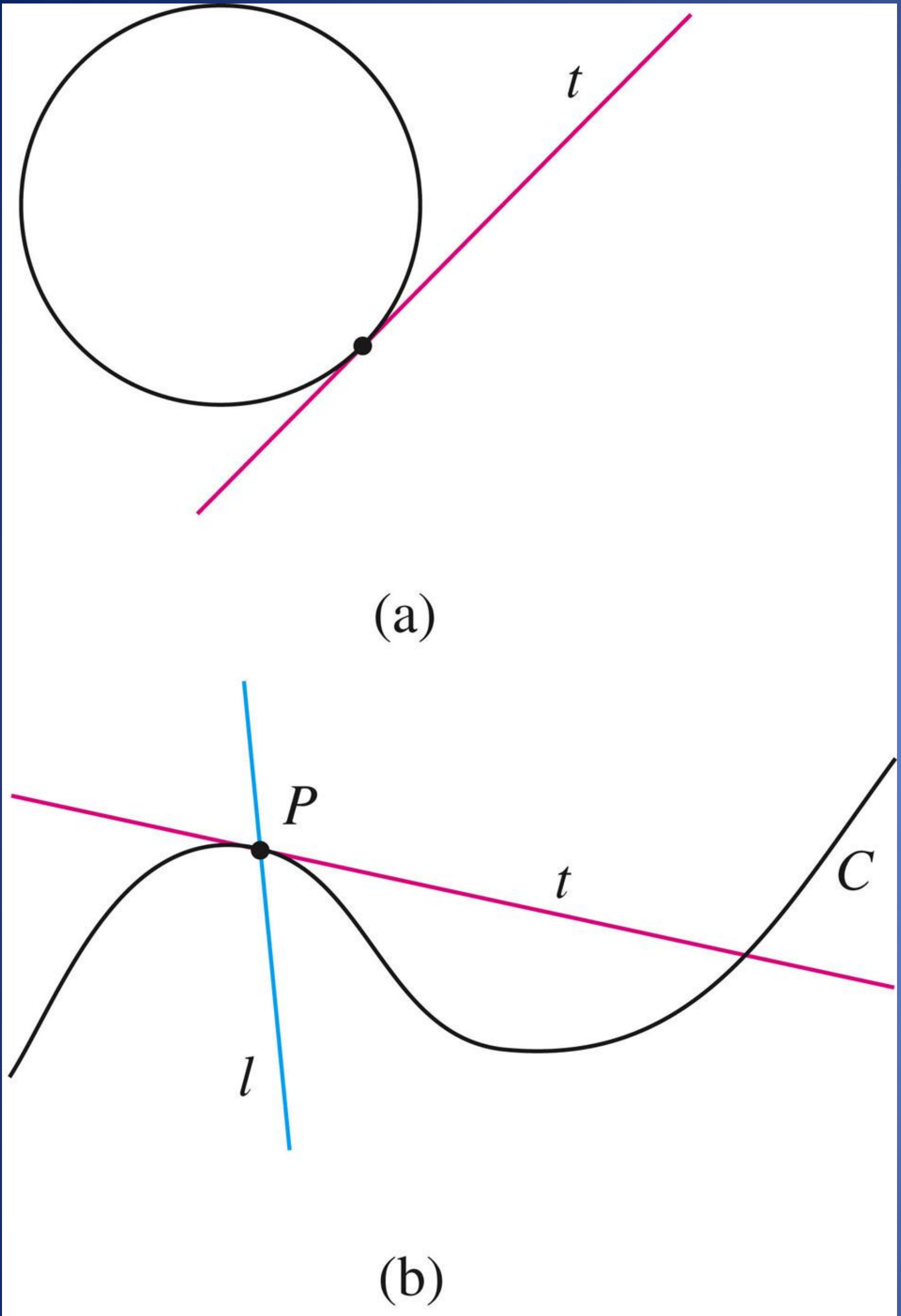


FIGURE 1

The word *tangent* is derived from the Latin word *tangens*, which means "touching."

Thus a tangent to a curve is a line that touches the curve.

In other words, a tangent line should have the same direction as the curve at the point of contact.

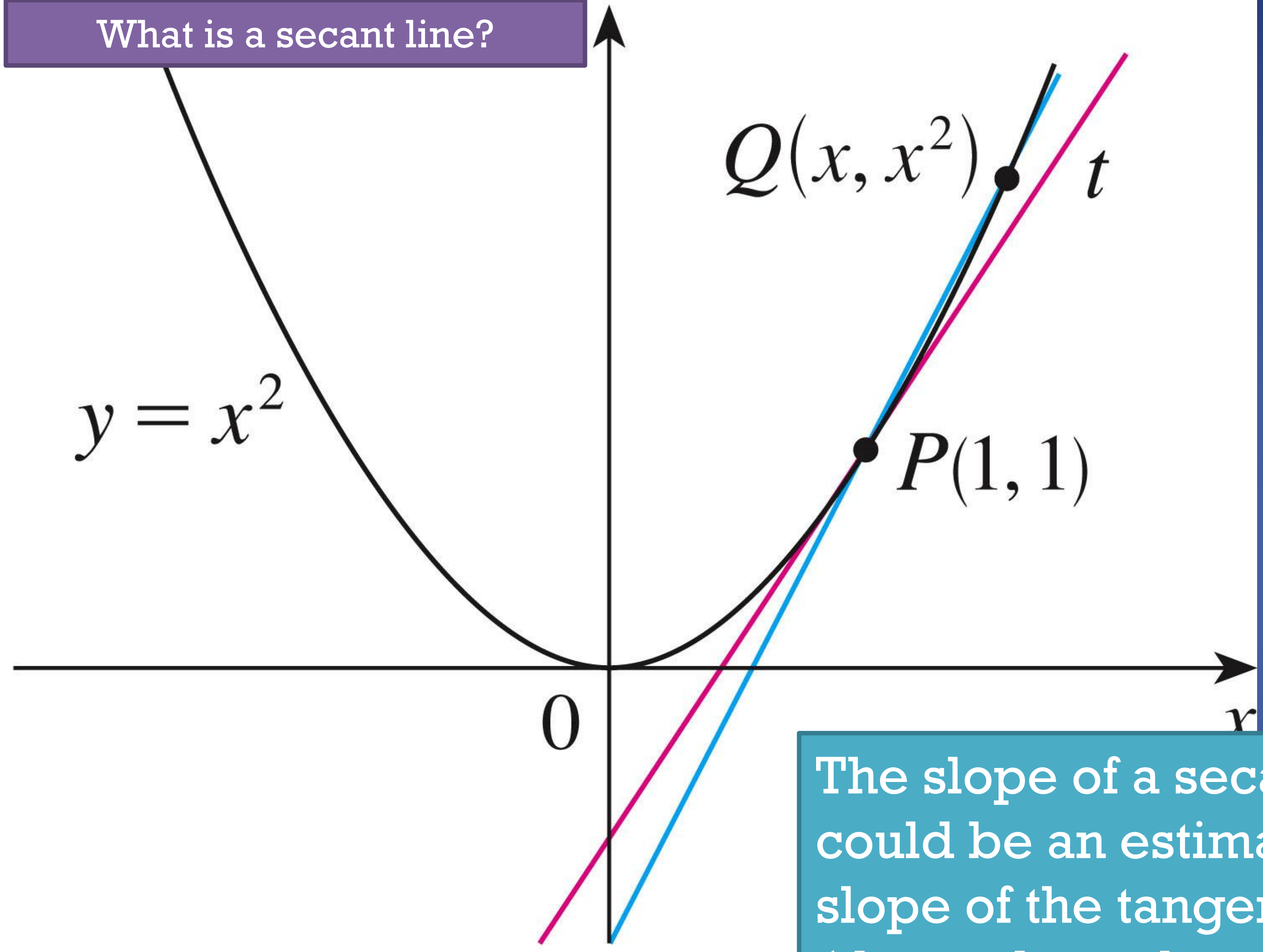
Example 1

- Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

- Solution:
- We will be able to find an equation of the tangent line t as soon as we know its slope m.

- The difficulty is that we know only one point, P, on t, whereas we need two points to compute the slope.

What is a secant line?



The slope of a secant line could be an estimate for the slope of the tangent line. (depends on the secant line)

FIGURE 2

Calculating the slope of the secant line PQ

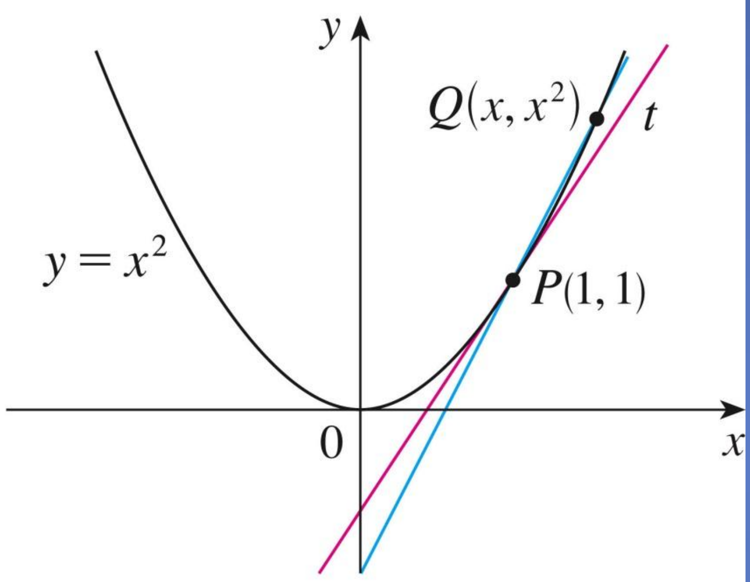


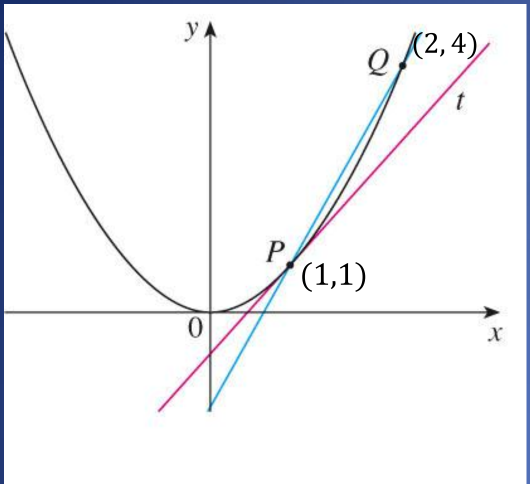
FIGURE 2

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

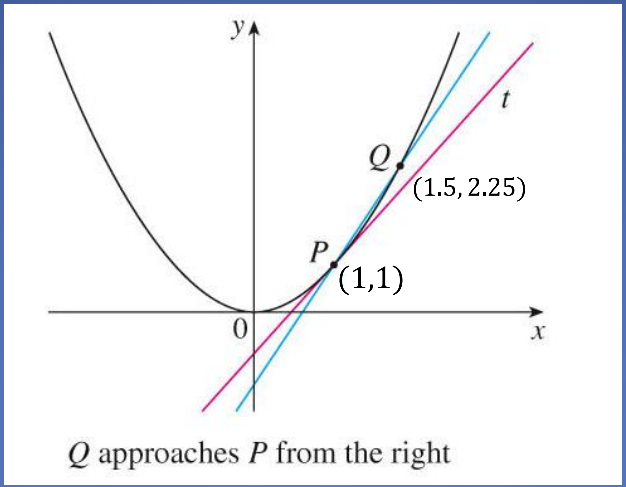
$$m = \frac{x^2 - 1}{x - 1}$$

Will need this later

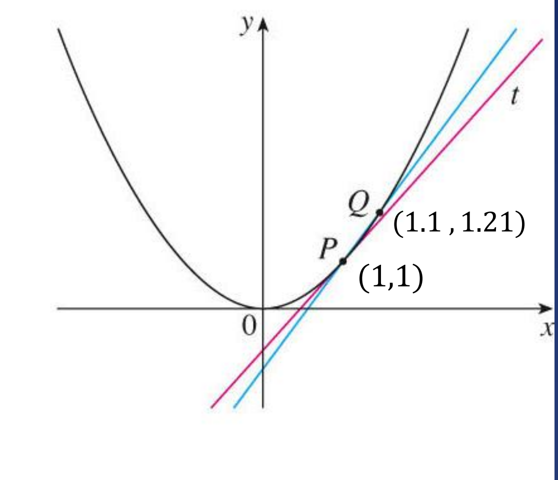
$Q = (2, 4)$



$Q = (1.5, 2.25)$

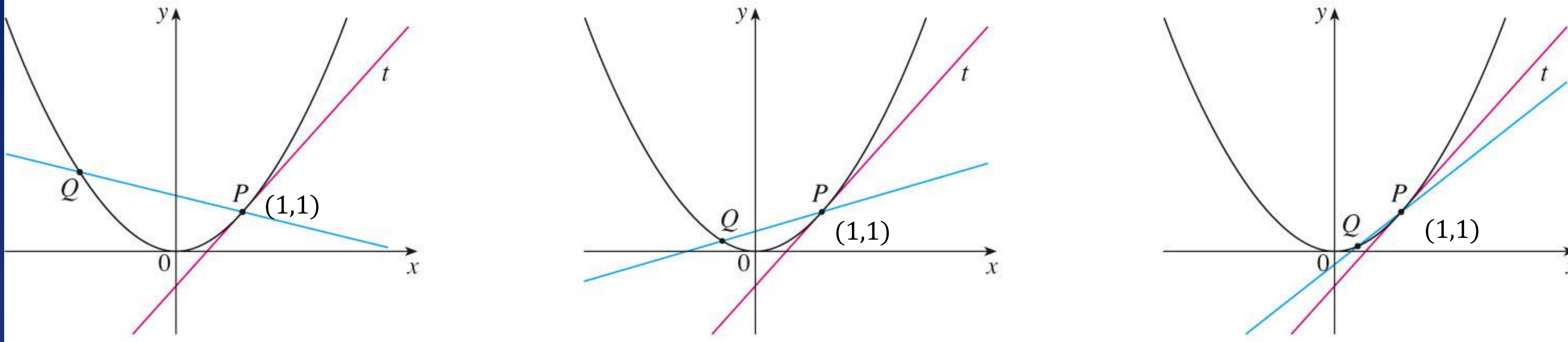


$Q = (1.1\,, 1.21)$



Note we will call this distance between the x values of P and Q, h

x value of Q As it approaches P	Slope
2	3
1.5	2.5
1.1	2.1
1.01	2.001



Q approaches *P* from the left

FIGURE 3

0	1
0.5	1.5
0.9	1.9
0.99	1.99

Show In Motion File
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The slope as a limit

cont'd

- This suggests that the slope of the tangent line t should be $m = 2$.

- We say that the slope of the tangent line is the *limit* of the slopes of the secant lines, and we express this symbolically by writing

-

$$\lim_{Q \to P} m_{PQ} = m$$

and

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$$

-

The Velocity Problem page 84-85

$s(t) = 4.9t^2$ is the model for free falling objects (neglecting air resistance) as disco
By Galileo.

Question: Suppose a ball is dropped from a tall building. Find the velocity of the ball after 5 seconds.

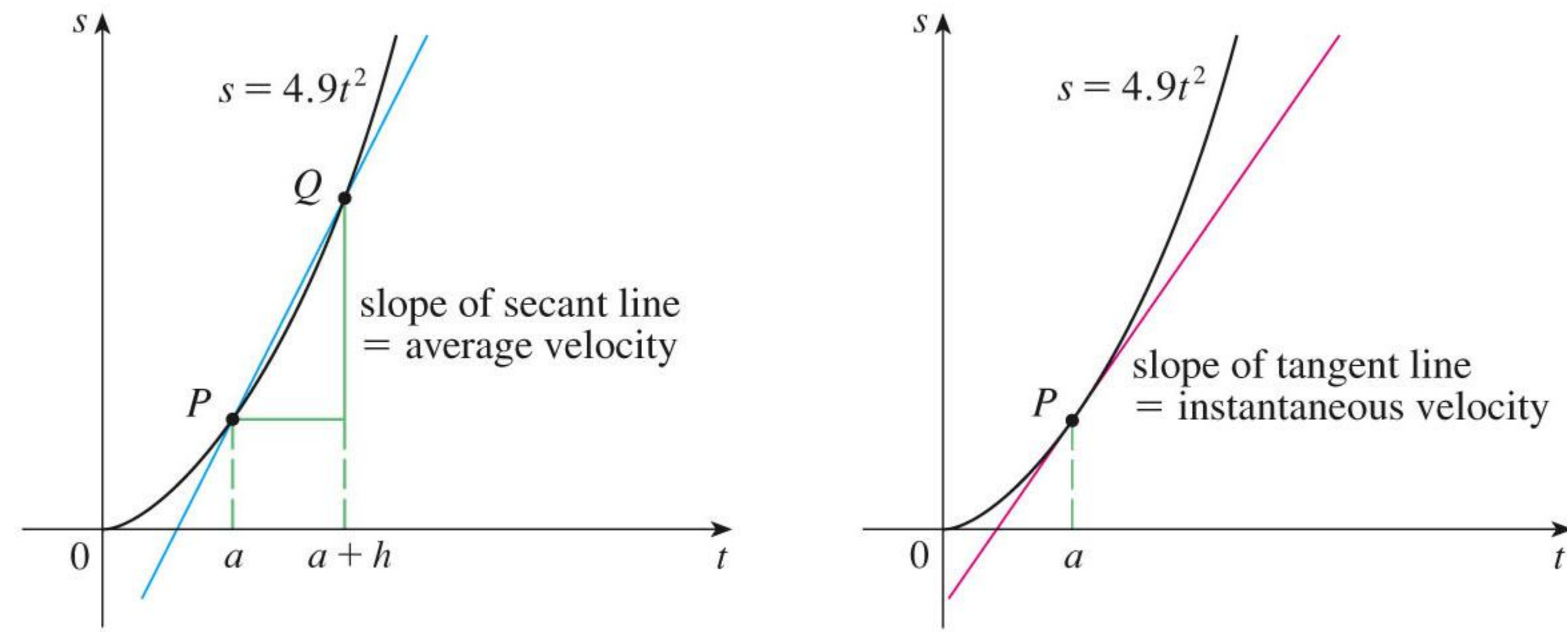


FIGURE 5

The Velocity AT 5 seconds would be the instantaneous velocity = slope of the tangent line at 5 seconds.
We don't know how to do this yet (well, we could take the limit and plug it into our fancy calculators). So for now we will estimate the instantaneous velocity by finding average velocity (secant lines) . Next we will do it by calculating limits.

Practice Worksheet

2.1 #s 1-7 odd

I’ve been told that it shouldn’t take too long.

2.2 The Limit of a Function

What is the difference between the statements

“$f(x) = L$ and "$\lim_{x \to a} f(x) = L$"

The Limit of a Function

- Let's investigate the behavior of the function f defined by $f(x) = x^2 - x + 2$ for values of x near 2.

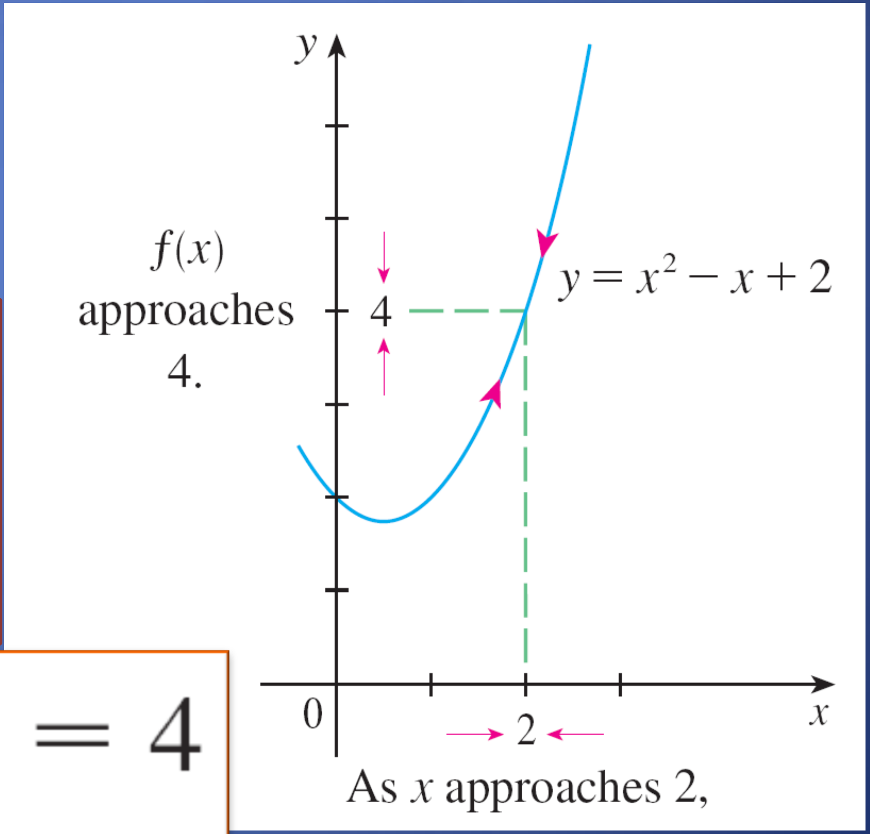
The Limit of a Function

- The following table gives values of $f(x)$ for values of x close to 2 but not equal to 2.

x	$f(x)$	x	$f(x)$
1.0	2.000000	3.0	8.000000
1.5	2.750000	2.5	5.750000
1.8	3.440000	2.2	4.640000
1.9	3.710000	2.1	4.310000
1.95	3.852500	2.05	4.152500
1.99	3.970100	2.01	4.030100
1.995	3.985025	2.005	4.015025
1.999	3.997001	2.001	4.003001

The Limit of a Function

- From the table and the graph of f (a parabola) shown in Figure 1 we see that when x is close to 2 (on either side of 2), $f(x)$ is close to 4.



"The limit of the function
$f(x) = x^2 - x + 2$
as x approaches 2 is equal to 4."

$$\lim_{x \to 2} (x^2 - x + 2) = 4$$

Figure 1

Unofficial Definition of a Limit

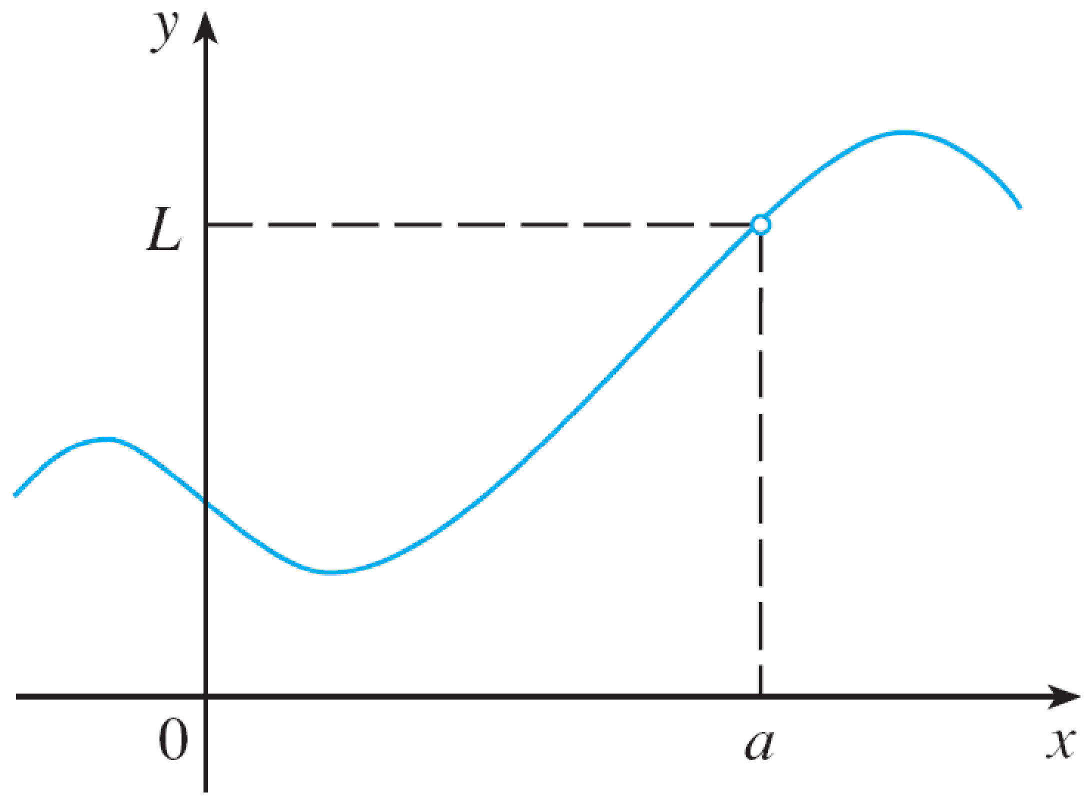
We say that $\lim_{x \to a} f(x) = L$ *if as* x *gets closer and closer to* a, $f(x)$ *gets closer and closer to* L

1 Definition We write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L"

if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a.



(c)

Limits are used to describe the behavior of a function's outputs "in the neighborhood" of a number x = a, where the function may or may not be defined.
In order for the limit to exist, the function's outputs must "approach" the same number from the left and the right of x = a.

You will explore limits numerically, graphically, and algebraically in this chapter.

Consider the functions $f(x) = \frac{x^3-8}{x-2}$.
Let's investigate its behavior near x = 2.

We need to approach 2 from the right and the left.

Compute the following on your paper

Examples 1 – 5 in your book
Read them.

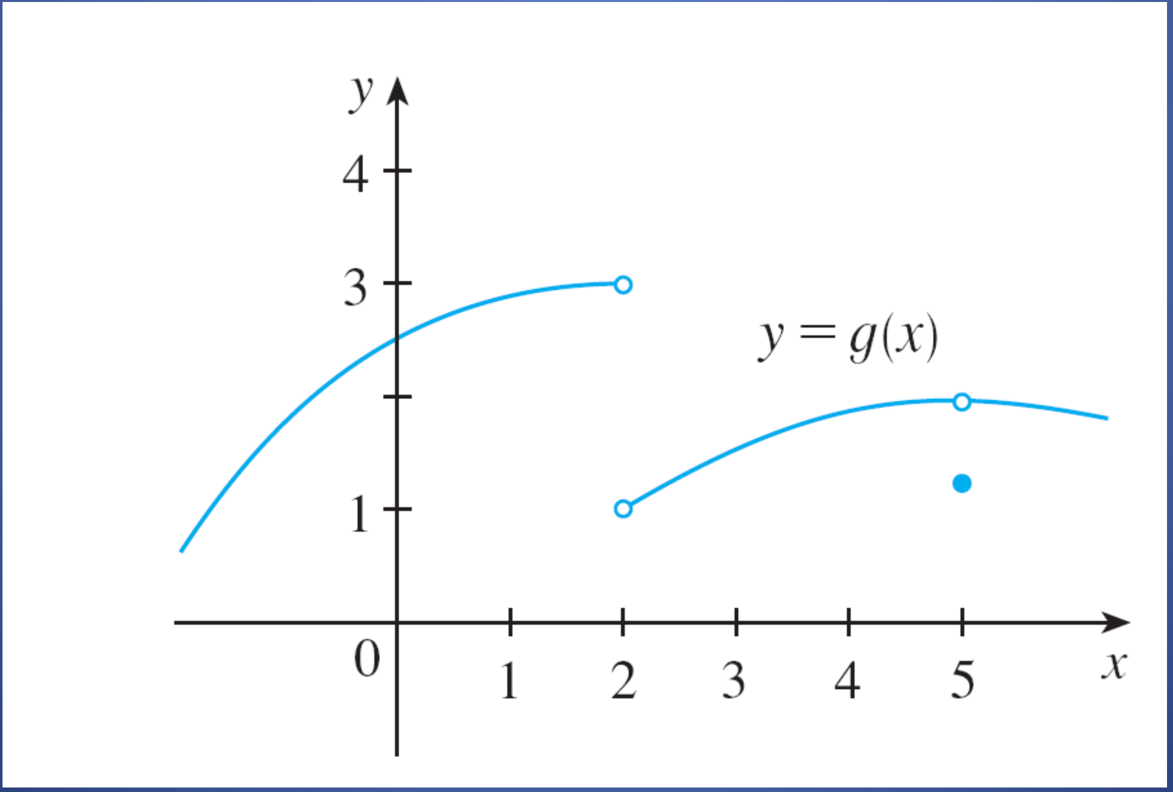
Left Hand and Right Hand Limits

(a) $\lim_{x \to 2^-} g(x)$

(b) $\lim_{x \to 2^+} g(x)$

(d) $\lim_{x \to 5^-} g(x)$

(e) $\lim_{x \to 5^+} g(x)$



$$\lim_{x \to a} f(x) = L$$

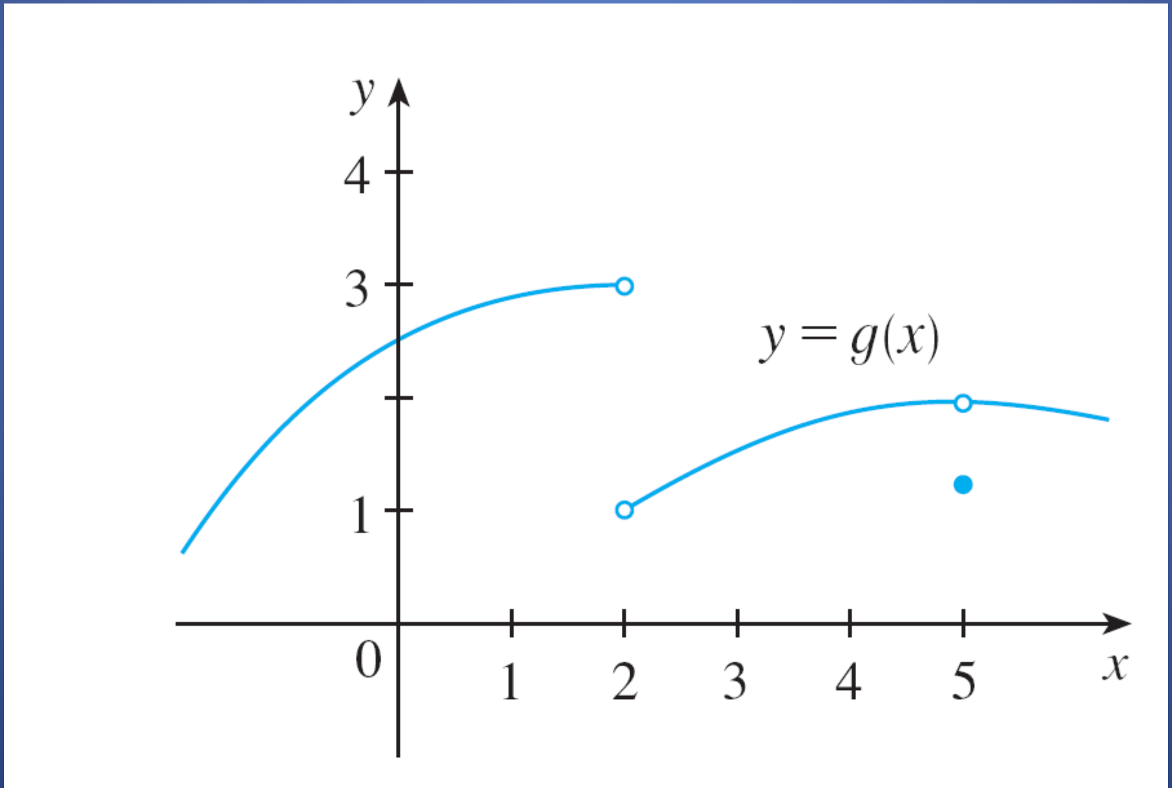
$$if\ and\ only\ if$$

$$\lim_{x \to a-} f(x) = L\ AND\ \lim_{x \to a+} f(x) = L$$

Left Hand and Right Hand Limits

(a) $\lim_{x \to 2^-} g(x)$ (b) $\lim_{x \to 2^+} g(x)$ (c) $\lim_{x \to 2} g(x)$

(d) $\lim_{x \to 5^-} g(x)$ (e) $\lim_{x \to 5^+} g(x)$ (f) $\lim_{x \to 5} g(x)$



Vertical Asymptotes Defined using Calculus

A Vertical asymptote is the line x=a if *f* has an infinite limit (possibly one sided) at *a*.